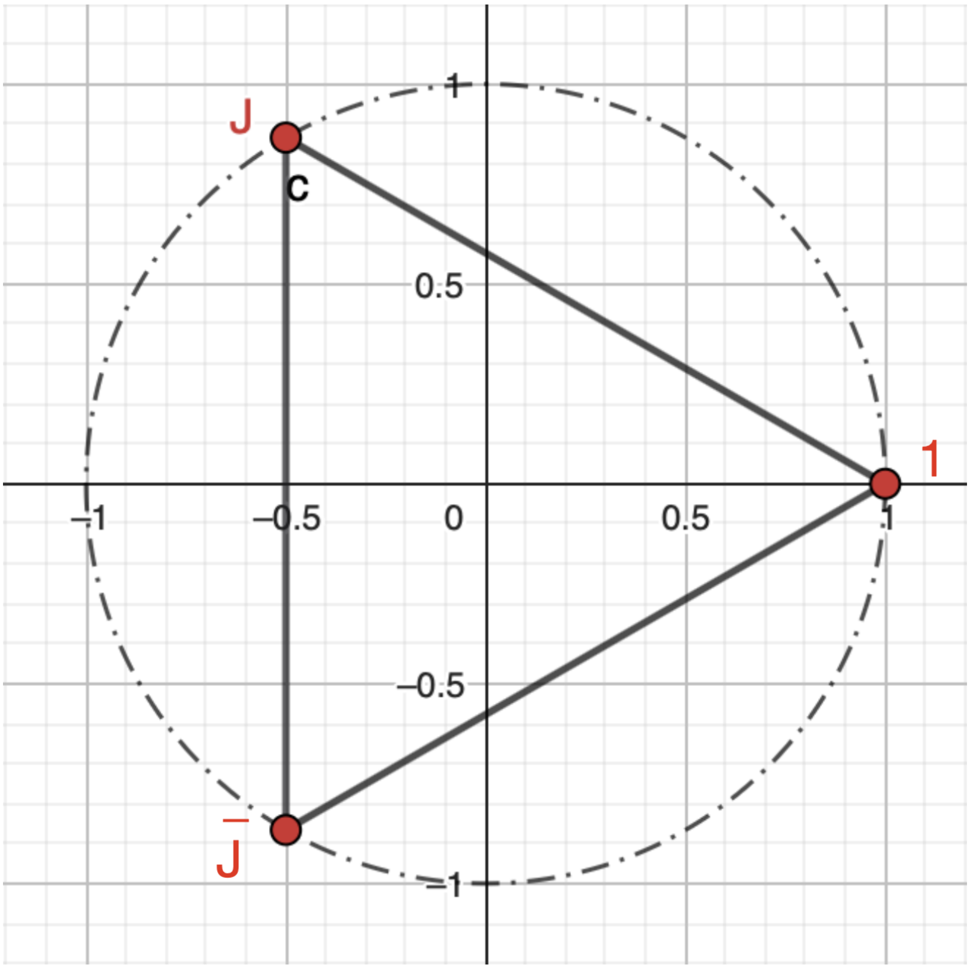
Digital Education

cs-411

# Learning About j

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Group A

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# Contents

# 1 Introduction

As part of the CS-411 Digital Education course at EPFL, our project involved designing a digital lesson in two different formats: **Instruction followed by Problem Solving** (**I-PS**) and **Problem Solving followed by Instruction** (**PS-I**). This dual approach enables us to study the effect of the order of instruction on students' learning outcomes.

The goal of the course and project is to explore the relationship between specific technological features and cognitive learning processes. The course also investigates the effectiveness of educational technologies through empirical studies, specifically examining, whether students genuinely benefit from using technology for learning.

Our lesson introduces students to the fundamentals of complex number mathematics with the aim of sparking curiosity, encouraging intuitive understanding and enhancing the learning experience. We want students to become familiar with the notation, properties, and visual representations of complex numbers.

A fundamental prerequisite for this lesson is knowledge of basic algebra, such as solving quadratic equations.

The goal is to use the specific complex number $j = \exp\left(\frac{2\pi i}{3}\right) = -\frac{1}{2} + \frac{\sqrt{3}i}{2}$ as a tool to explore geometric interpretations in the complex plane and on the unit circle, and to examine the properties of complex numbers. This focus is intended to engage learners with the mathematical properties and visual representations of complex numbers.

## 2 Learning Goals

The primary aim of the lesson is to help students solve equations involving the imaginary unit $i$ and to give them a solid understanding of the fundamental properties of complex numbers and how to manipulate them. The lesson also aims to promote transfer learning by encouraging students to apply this knowledge in new contexts.

To this end, the **Instruction** part of the activity includes guided explanations and complex number operations. By the end of the lesson, students should have mastered these operations, grasped their visual representations, and their relevance in broader mathematical contexts.

Therefore, by the end of the lesson, students should be able to:

1. Identify whether the solution of an equation is real or complex.
2. Recognise and compute the conjugate of a complex number.
3. Solve basic equations involving complex numbers.
4. Recognise when and why to use the algebraic form, $z = a + bi$, or the trigonometric form, $z = \cos(\theta) + i \cdot \sin(\theta)$, of a complex number.
5. Represent complex numbers visually in the complex plane.

# 3 Lesson Design & Activities

For our lesson, we collaborated with M. Christophe Haton-Roebuck, a high school mathematics teacher at Bugnon Ours, in Lausanne, who teaches a second-year class specialising in maths and physics. M. Haton-Roebuck showed great interest in our project. We agreed to base our lesson on the topic of complex numbers, as his students had just begun exploring this concept, which made them ideal participants for this activity.

Together, we designed a one-hour activity aligned with his curriculum. A key aim was to engage the teacher in a way that would contribute to his professional development and expose him to alternative teaching methods that could make challenging topics more approachable and inspiring for students. Mr. Haton-Roebuck appreciated this aspect and even requested the materials to use with future classes.

The lesson's goal is to guide students toward discovering the complex number $j$, without explicitly revealing that this was their objective. Through progressively challenging tasks and quizzes, we designed a conceptual journey — *'a quest for $j$'.*

Students were divided into two groups with different instructional sequences: one followed Problem-Solving then Instruction (PS-I), the other Instruction then Problem-Solving (I-PS). The lesson is structured as follows:

1. **Presentation:** Brief introduction to the activity and its format.
2. **General Questionnaire:** Collected background information (e.g., gender, age).
3. **Pre-Test** (10min)**:** Assessment of general understanding of basic mathematics.
4. **Core Lesson** (45min)**:**
   - **Instruction** (15min)**:** Interactive introduction to complex numbers using `GoeGebra`.
   - **Problem Solving** (30min)**:** Activity designed to promote exploration and highlight learning gaps (Productive Failure).
5. **Post-Test** (10min)**:** Evaluate students' final understanding and the impact of the lesson.
6. **Debriefing:** Brief in-class explanation on how to solve the main problem and presentation of the interesting properties of the complex number $j$.

### *Pre-Test*

Students completed four questions in approximately 10 minutes, covering number sets, fractions, quadratics, and transfer knowledge problems. It assessed students' basic mathematics skill. Two questions were randomly selected from a pool of equivalent items to reduce the risk of cheating while maintaining consistent difficulty. This assessment identified initial knowledge gaps among students and previewed concepts used later.

### *Problem Solving Activity*

All students addressed the equation $j^3 = 1$ and two follow-ups: (1) which solutions are closer to each other, and (2) which is closest to the origin. Though simple in appearance, solving the problem required recognizing the need for complex numbers. This task was intentionally designed to trigger Productive Failure — a pedagogical approach that embraces initial struggle as a pathway to deeper understanding.

This activity targeted three Learning Goals:

- Identifying whether the solutions are real or complex.
- Recognizing the conjugates of complex numbers (two solutions are conjugate).
- Solving basic equations involving complex numbers.

The follow-up questions also evaluated students' ability to visualize complex numbers in the complex plane (Learning Goal 5).

### *Instruction*

This segment introduced complex numbers through seven topics: definition, geometric interpretation, operations (addition, subtraction, multiplication, complex conjugate), and trigonometric form. Using `GeoGebra`, we provided visualizations, interactivity, and embedded quizzes to reinforce understanding.

The visual and exploratory nature of this instruction supported key learning objectives : recognizing and computing conjugates, solving simple equations involving complex numbers, and visualizing complex numbers in the complex plane.

### *Post-Tests*

To conclude the session, all students took a post-test consisting of two exercises. The first asked students to place given complex numbers on the complex plane, assessing their understanding of visual representation (Learning Goal 5). The second problem required solving an equation using complex numbers, assessing their grasp of when and how to apply complex solutions (Learning Goal 1).

### *Easter Eggs*

To add an element of surprise and reinforce engagement, we included several *'Easter eggs'* throughout the lesson. The complex number $j$ was deliberately concealed — either as a solution to equations or as a number to place on the complex plane — without explicitly naming it. This approach allowed us to playfully integrate $j$ into the learning process and highlight its interesting mathematical properties. During the final debriefing, we revealed this *'hidden quest'*, which was well received by both students and the teacher.

# 4 Experimental Design

The core objective of our experimental design is to determine whether one instructional sequence — **PS-I** (Problem Solving followed by Instruction) or **I-PS** (Instruction followed by Problem Solving) — offers superior benefits in terms of student learning. Our protocol was therefore structured to compare the effects of these two teaching methods by measuring learning gains and analysing students' reasoning processes across conditions.

To do so, we designed a three-part protocol: a **Pre-Test**, a **Learning Activity** (composed of the *Instruction* and *Problem-Solving*, the order depending on condition), and a **Post-Test**. The content was aligned with five predefined Learning Goals (LGs) covering key concepts related to complex numbers. Questions were crafted to allow us to trace progress on each LG throughout the session. While some LGs could be evaluated both before and after the activity (e.g., solving equations — LG1 and LG3), others were only measurable post-instruction (e.g., use of conjugates or visualisation in the complex plane — LG2, LG4 & LG5).

The **Pre-Test** assessed students' prior knowledge (especially algebraic manipulation and equation solving), enabling us to verify group equivalence and later calculate both absolute and relative learning gains. The **Post-Test** then evaluated conceptual understanding and application across all LGs. Additionally, the students' draft sheets were analysed qualitatively to capture the types of reasoning attempts made during the Activity phase.

To interpret the results, we employed both quantitative and qualitative analyses:

- A one-way **ANOVA** — to test whether the dependent variable learning gains is significantly affected by the independent variable condition (groups).
- **Chi-squared** tests — to explore the association between independent variable reasoning strategies (mapped to LGs) and dependent instructional condition.

To ensure data reliability, we filtered out outliers and carefully examined the completion time, answer correctness, and reasoning quality for each task. The results were interpreted in light of the protocol design, with attention to whether instructional order promotes better engagement with the conceptual challenges inherent in complex number problem-solving.

# 5 Implementation

Our experimental lesson was delivered via the `Moodle` learning management system, chosen for its modular structure, interactivity, data logging, and ease of use. Instructional content, assessments, quizzes and data collection tools were fully integrated, with dynamic visualizations provided through embedded `GeoGebra` simulations.

The activity was administered to a class of 15 students at the Bugnon Ours High School, in Lausanne. We were set up in a computer room, where each student had their own workstation to complete the activity individually and minimize distractions.

## 5.1 Experimental Setup

The experiment is structured to compare two instructional sequences: **PS-I** (Problem Solving before Instruction) and **I-PS** (Instruction before Problem Solving). Students were randomly assigned to the two groups based on their chosen seating.

The sequence followed a four-phase:

1. Pre-Test  2. Instruction (I-PS group) / 3. Problem Solving Activity (PS-I group)

2. Problem Solving Activity (I-PS group) / 3. Instruction (PS-I group)  4. Post-Test

Each phase was implemented using `Moodle`'s *Quiz* module, with `GeoGebra` used for interactive concept exploration for the Instruction. Students also received draft sheets to record their reasoning, process later collected for qualitative analysis. All responses and logs were recorded for further analysis.

## 5.2 Four-Phases

### *Pre-Test*

The **Pre-Test** included four questions designed to assess baseline skills :

Q1 : **Classification of Numbers (Randomized MCQ)** – Implemented using `Moodle`'s *Random* question feature, this question presented students with one of four predefined lists of numbers. Students have to classify these numbers by selecting all appropriate number sets (e.g., rational, real).

Q2 : **Simplification of a Fraction (Numerical Response)** – Configured using the *Numerical* question type, students are required to simplify a fractional expression and enter the final value directly into a response box.

Q3 : **Solving a Quadratic Equation (Multiple Solution Entry)** – Students are presented with a randomly selected quadratic equation of the form $ax^2 \pm bx \pm c = 0$, generated from a question bank. Solutions were entered using a *Multiple Solution Equation* setup to fill in the two solutions of the equation.

Q4 : **Circuit Analysis Problem (Embedded Numerical Entry)** – Real-world application problem involving a resistive circuit. Students were given partial values and equations, then required to form and solve a system of equations. The response is collected using the *Embedded Answers (Cloze)* format.

### *Problem Solving Activity*

Common to both groups but delivered in different sequence, the task included a two-part question implemented in a single `Moodle` quiz item. :

P1 : **Equation Solving** – Students have to solve the equation $z^3 = 1$, entering all three solutions using the *Cloze* format to ensure structured, numeric input. However, they are not given the information that the solutions are complex numbers.

P2 : **Conceptual Justification** – The follow-up questions prompted students to explain the relationships between their solutions (which are closest to one another, which is closer to the origin) in free-text fields embedded within the same *Cloze* question. This component was used to assess depth of understanding and reasoning.

### *Instruction*

Also common to both groups, the **Instruction** module covered seven topics each hosted in a `Moodle` Quiz. Each lesson focused on one foundational concept in complex numbers:

1. Algebraic Form ($z = a + i \cdot b$)
2. Geometric Representation in the complex plane
3. Addition
4. Subtraction
5. Complex Conjugate
6. Multiplication
7. Trigonometric Form ($z = \cos(\theta) + i \cdot \sin(\theta)$)

Each section included:

- A brief theoretical explanation with formal definitions.
- A `GeoGebra` interactive visualization embedded using a `GeoGebra` plug-in. These features are interactive and students are encouraged to play around with a cursor to make the graphic visualization change and allow them to understand graphically the concept.
- A final quiz question (Q&A) designed to reinforce the concept through active engagement with the visualization.

### *Post-Test*

The **Post-Test** assessed retention and transfer of knowledge to novel contexts:

E1 : **Complex Plane Visualization (Drag and Drop Markers)** – Students were provided with six complex numbers and a coordinate grid as a background image. Using `Moodle`'s `Drag and Drop Markers` question type, students placed the numbers at their correct locations on the complex plane.

E2 : **Impedance Circuit Problem (Numerical Transfer Task)** – Similar to the Pre-Test's final question, this task required solving an electrical impedance problem. However, this time, only minimal contextual clues were provided. The question was structured using the *Cloze* format, with fields for each key computational step. Crucially, it was not disclosed that the expected solution could be complex — thus assessing the student's ability to transfer mathematical understanding to new domains.

## 5.3 Data Collection

All quiz responses, including timestamps, selected answers, and numerical entries, were logged via `Moodle` and exported for analysis. `GeoGebra` interactions were not logged directly but inferred through performance and observations.

The full implementation included:

- 4 `Moodle` quizzes (Pre-Test, Instruction, Problem Solving, Post-Test)
- 6 embedded `GeoGebra` visualizations
- 2 randomized question banks
- 4 question types: `Multiple Choice`, `Numerical`, `Cloze`, `Drag and Drop`

All components were designed for replicability, automation, and streamlined analysis and all written justifications, from the draft sheets, were analysed by hand for further data to analyse.

# 6 Participants, Data & Analysis

## 6.1 Participants, Group Repartition & Pre-Test Analysis

Fifteen students participated in the study and were evenly divided into two groups: **PS-I** (7 students) and **I-PS** (8 students). While the sample size was smaller than anticipated, students were specialised in advanced mathematics and are currently studying complex numbers which provided a suitable context for the experiment.

| | count | mean | std | min | 25% | 50% | 75% | max |
|---|---|---|---|---|---|---|---|---|
| condition | | | | | | | | |
| IPS | 8.0 | 4.88 | 2.97 | 1.50 | 2.38 | 4.50 | 6.75 | 10.0 |
| PSI | 7.0 | 3.18 | 2.93 | 0.25 | 1.00 | 1.75 | 5.38 | 7.5 |

Figure 1: Grade statistics of the Pre-Test depending on the condition

The **Pre-Test** results showed that the I-PS group performed better than the PS-I group, with mean scores of 4.88/10 and 3.18/10, respectively. Median scores reinforced this difference: 4.5 for I-PS and 1.75 for PS-I. Although variability was high in both groups (SD $\approx$ 2.95), the I-PS group appeared to have stronger prior knowledge.

Overall, the I-PS group answered more accurately and confidently than the PS-I group. To understand the range of scores, it is important to note that scores were assigned as follows:

Incorrect answers $= -1$ No answer $= 0$ Partially correct answers $= [0; 2.5]$

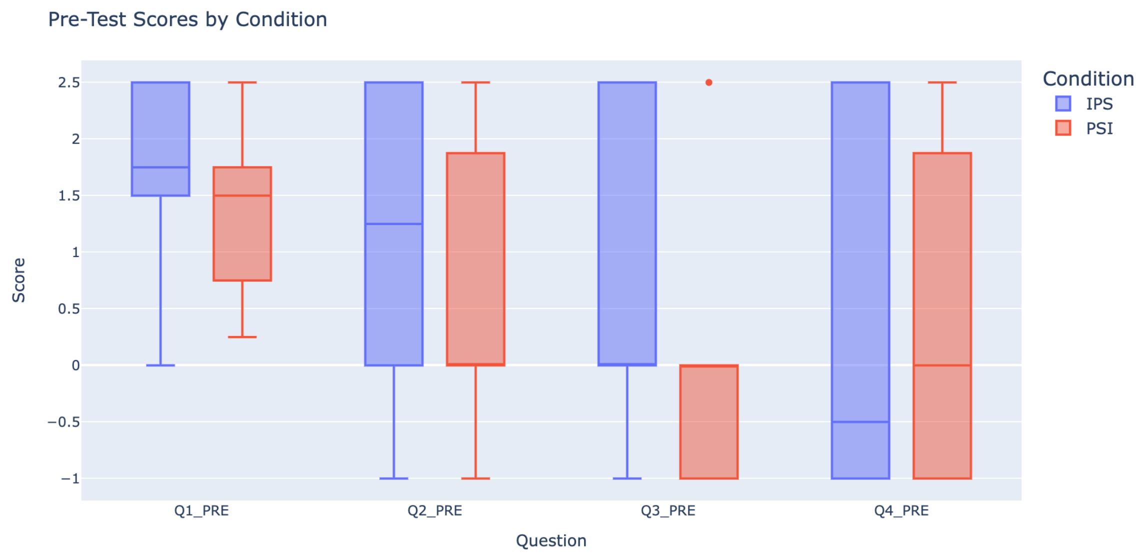


Figure 2: Pre-Test Question Scores by Condition

In Questions 1 and 2, results were comparable across groups, though I-PS performed slightly better, with a slightly higher average. The positive mean here demonstrates not only correct answers, but also fewer incorrect guesses.

However, in Questions 3 and 4, the I-PS group performed significantly better. Particularly in Q3, a question requiring students to solve a second-degree equation, PS-I students struggled notably, despite they presumably learned this previously, which is why these are surprising results. These findings highlight the need to consider initial disparities when analysing **Post-Test** outcomes.

Even though most students in the PS-I group still tried to solve the question rather than skip it, it suggests students in the I-PS group had a better understanding of the material for more complex questions. We will bear these results in mind when interpreting the **Post-Test** results, as they may reflect pre-existing differences rather than the effects of the study.

## 6.2 PS-I vs. I-PS : Pre-Test / Post-Test Comparison

### 6.2.1 *Effect of Condition on Learning Gain*

We analysed absolute (*Post-Test* − *Pre-Test*) and relative (normalised by initial performance) learning gains to evaluate instructional effectiveness.


Figure 3: Learning Gain and Relative Learning Gain by Condition

The **Violin plots** show that PS-I resulted in more consistent gains, whereas I-PS outcomes were highly variable, with several students scoring lower on the **Post-Test** than on the **Pre-Test**. Normalised gains also favored PS-I, supporting the hypothesis that prior problem engagement fosters more reliable learning.

To assess statistical significance, we ran an **ANOVA** test using the model: Learning Gain $\sim$ C (condition). Results showed no significant effect ($p = 0.418$, $F \approx 0.7 <$ critical $F_{0.05} = 4.67$), likely due to the small sample size.

A power analysis confirmed this limitation: with an effect size considered moderate, the test power was only 14.6%, meaning a high likelihood (85.4%, as $\beta = 1 - \text{power} = 0.854$) of failing to detect a true effect on a sample of $n = 15$ (type II error). Therefore, this justifies the weak presence of the results previously obtained with the **ANOVA**.

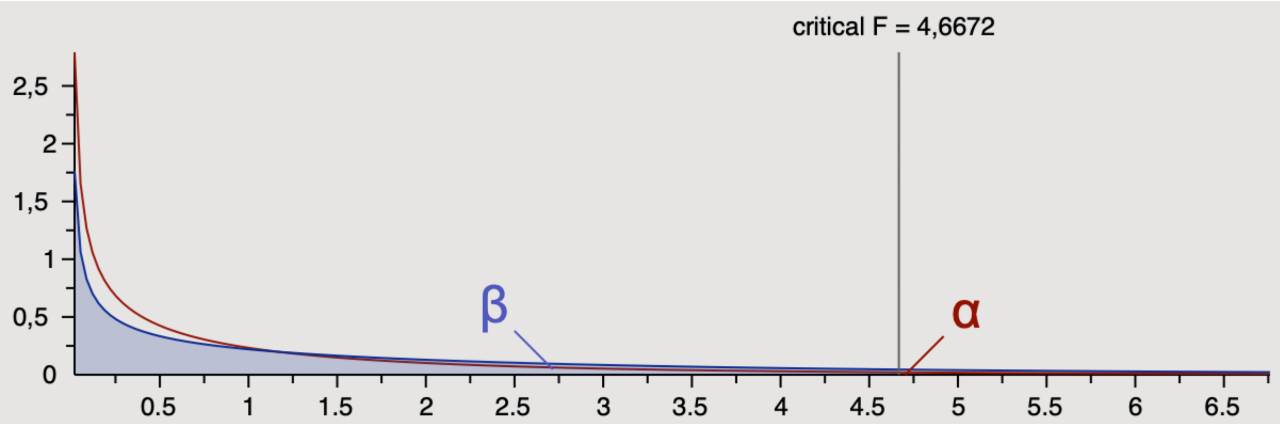


Figure 4: Power Analysis of Learning Gain by Condition

#### 6.2.2 Effect of Reasoning Attempts on Condition

To complement the **ANOVA** analysis, we used **Chi-squared** tests to examine the relationship between reasoning types and instructional condition. Of five identified strategies, only one — *recognizing the existence of a complex solution* — showed statistically significant association with condition ($\chi^2 = 4.57$, $p = 0.0325$). Six of seven PS-I students used this reasoning, compared to one of eight in I-PS. This supports the idea that PS-I promotes early engagement with core concepts.

Two other strategies ( *"finding another expression for z"* and *"for 1"*) approached significance threshold ($p \approx 0.076$), indicating possible exploratory tendencies in PS-I, students may have been more inclined to explore alternative representations. The other strategies ( *"attempting to solve the equation"* and *"geometric reasoning"*) showed no significant differences between groups ($p = 0.5892$ and $p = 0.4450$ respectively), suggesting some reasoning approaches are less sensitive to instructional sequence.

In summary, the Chi-squared analysis highlights that the order of instruction can influence the kind of reasoning students are willing to try, particularly in the earliest stages of conceptual understanding.

### 6.2.3 Pre-Test vs Post-Test : focus on Learning Objectives

Student work was analysed against the five Learning Goals.

**Learning Goal 1 : Identify whether the solution of an equation is real or complex.**

Only four students (1 I-PS, 3 PS-I) succeeded, despite this being prior knowledge, these results are somewhat surprising. The low success rate suggests difficulty applying conceptual understanding under test conditions.

**Learning Goal 2 : Recognise and compute the conjugate of a complex number.**

All but one student in the I-PS group and all but two students in the PS-I group achieved this, reflecting the emphasis in instruction and test design. Students were given clear definitions and multiple opportunities to practise identifying and calculating conjugates, which seems to have supported their understanding quite effectively. The few students who did not succeed may have struggled with symbolic manipulation or paying attention to detail, but the concept was generally well understood.

**Learning Goal 3 : Solve basic equations involving complex numbers.**

No student met this objective, despite its coverage in prior coursework. Even though a few I-PS students answered correctly in the **Pre-Test**, this did not translate into consistent **Post-Test** success.

**Learning Goal 4 : Recognise when and why to use the algebraic form, $z = a + bi$, or the trigonometric form, $z = \cos(\theta) + i \cdot \sin(\theta)$, of a complex number.**

Only two PS-I students achieved this. Their success suggests that encountering the problem first fostered a better grasp of when transformations are needed — something not observed in the I-PS group despite prior instruction.

**Learning Goal 5 : Represent complex numbers visually in the complex plane.**

All but one student in each group plotted most of the points correctly. This was a new concept introduced during instruction and was not part of the **Pre-Test**, suggesting good uptake from the lesson.

| | LO1_POST | LO2_POST | LO3_POST | LO4_POST | LO5_POST | number_students |
|---|---|---|---|---|---|---|
| **condition** | | | | | | |
| **IPS** | 1 | 7 | 0 | 0 | 7 | 8 |
| **PSI** | 3 | 5 | 0 | 2 | 6 | 7 |

Figure 5: Summary of Post-Test results

# 7 Conclusions & Discussion

## 7.1 Discussion

While **Post-Test** results revealed trends in achieving the Learning Goals, they must be interpreted alongside the **Pre-Test** data.

It was unexpected that very few students achieved Learning Goal 1 — *Identifying whether an equation's solution is real or complex* — despite relatively solid Q1-PRE scores (means of 1.75 and 1.39 for I-PS and PS-I, respectively).

In contrast, strong performance on Learning Goal 2 — *Computing the conjugate of a complex number* — are consistent with the low Q2-PRE scores, suggesting effective learning during the **Instruction** phase.

As for Learning Goal 4 — *Choosing between algebraic and trigonometric forms* — low prior knowledge in both groups (Q4-PRE mean of 0.44 in I-PS) and stronger **Post-Test** results in the PS-I group suggest that **Productive Failure** may have fostered deeper understanding.

Beyond the Learning Objectives, we also analysed students' **reasoning attempts** to better understand how the condition influenced their approach to the problem. Notably, recognizing the existence of a complex solution was significantly more common in the PS-I group. This indicates that tackling the problem before instruction may prompt intuitive engagement, which is consistent with the idea of Productive Failure

Other exploratory strategies (e.g., re-expressing $z$ or 1) followed similar, though non-significant, trends. In contrast, strategies like solving the equation or geometric reasoning appeared equally in both groups, possibly reflecting general habits rather than effects of instructional order.

## 7.2 Conclusion

This study examined how instructional order influences the learning of complex numbers, comparing Problem Solving followed by Instruction (PS-I) with Instruction followed by Problem Solving (I-PS).

While both sequences have merits, PS-I appears to promote more consistent gains, particularly in tasks requiring conceptual transfer. Despite the limited sample size and lack of statistical significance, qualitative patterns suggest that PS-I supports deeper engagement and exploratory thinking, reinforcing the potential benefits of introducing challenges before formal instruction.

# 8 Annexes

Links to our Moodle page for each part referenced in the previous sections :

1. General information survey :

   https://www.cs411-moodle.com/mod/questionnaire/view.php?id=1715

2. Pre-Test quiz :

   https://www.cs411-moodle.com/mod/quiz/view.php?id=1690

3. Problem Solving Activity :

   https://www.cs411-moodle.com/mod/quiz/view.php?id=1693

4. Introduction :

   https://www.cs411-moodle.com/mod/quiz/view.php?id=1692

5. Post-Test quiz:

   https://www.cs411-moodle.com/mod/quiz/view.php?id=1691

Jupyter Notebook :

https://colab.research.google.com/drive/1GSTgzKihFNWf-N8HihYF2YoMoibJvmMh?usp=sharing